



Potters Gate C of E Primary School

Calculation Policy 2023-2024

CALCULATION POLICY

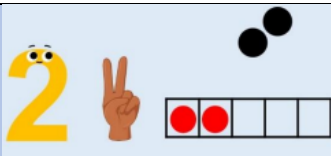
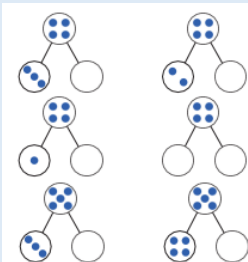
Over the years much has changed in the teaching and learning of maths. The calculation methods used by children today in many cases differ from those used by adults when they were at school. This can cause anxiety, with parents and carers unsure whether they should teach children particular methods.

The purpose of this booklet is to provide guidance and information about the types of calculation methods that the children at Potters Gate are being taught and are using from reception up to Year 6.

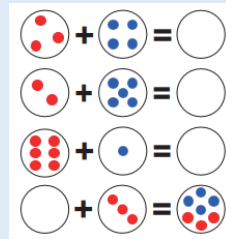
This policy lays out the expectations for both mental and written calculations for the 4 number operations and has been created to support the teaching of a mastery approach to mathematics. This is underpinned using models and images that support conceptual understanding and this policy promotes a range of representations to be used across the year groups. Mathematical understanding is developed through the use of representations that are first of all concrete (e.g. Dienes apparatus and place value counters), and then pictorial (e.g. bar models) to facilitate abstract working (e.g. standard written methods). This policy is a guide through an appropriate progression of representations and if at any point a pupil is struggling with the abstract, they should revert to familiar pictorial and/or concrete materials/representations as appropriate.

Although this policy sets out the main methods of mental and written calculations to be taught, it has been appended with a list of recommendations and effective practice teaching ideas aimed at informing and enhancing teaching across all the primary phases. Many of these ideas come from the NCETM's Calculation Guidance document (published October 2015) which is intended to sit alongside a school's calculation policy.

Addition	
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Year Group	Strand/Topic	Representation	Key Idea																		
Reception	Perceptual Subitising Composition	 	A key development underpinning the ability to subitise. Perceptual subitising is when pupils can recognise the quantity of items in groups up to 5 without counting each item. This is a mathematical structure that underpins all addition situations. Numbers can be understood in a practical /roleplay exploration and through pictorial stories. “How many birds are sitting on the fence?” How many birds are in the tree? How many birds altogether? What if 1 bird flew on the fence, how many birds can you see?																		
Reception	Perceptual Subitising Part – Part – Whole	<table><tr><td></td><td>0</td><td>zero</td></tr><tr><td>•</td><td>1</td><td>one</td></tr><tr><td>• •</td><td>2</td><td>two</td></tr><tr><td>• • •</td><td>3</td><td>three</td></tr><tr><td>• • • •</td><td>4</td><td>four</td></tr><tr><td>• • • • •</td><td>5</td><td>five</td></tr></table> 		0	zero	•	1	one	• •	2	two	• • •	3	three	• • • •	4	four	• • • • •	5	five	A key development underpinning the ability to add is subitising. Perceptual subitising is when pupils can recognise the quantity of items in groups up to 5 without counting each item. This is a mathematical structure that underpins all addition situations. Numbers can be understood in terms of their parts; understanding that the parts are part of a larger collection.
	0	zero																			
•	1	one																			
• •	2	two																			
• • •	3	three																			
• • • •	4	four																			
• • • • •	5	five																			

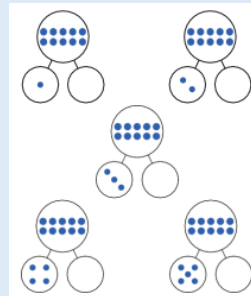
Conceptual Subitising



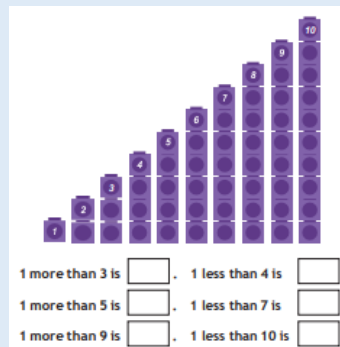
Composition of 5



Composition of 10



Adding 1, 1 More



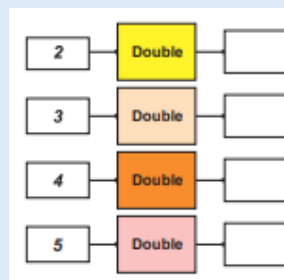
Pupils are able to recognise a quantity by combining groups that have not needed to be counted. Pupils may see 5 items as 3 items and 2 items.

Pupils are able to demonstrate all possible whole number compositions of 5, including recognising and showing 5 on a five frame and using a number bond diagram.

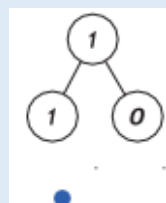
Pupils are able to demonstrate all possible whole number compositions of 10, including recognising and showing 10 on a ten frame and using a number bond diagram.

Pupils relate adding 1 to 1 more than the starting number.

Doubles



Adding zero

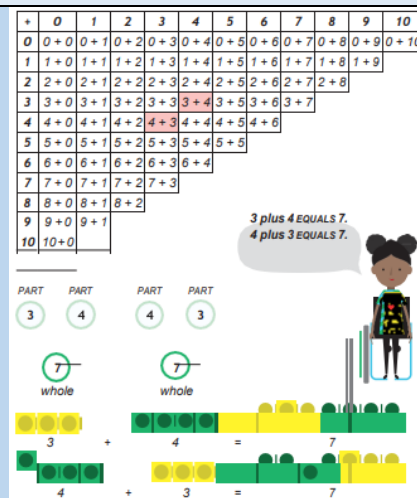


Pupils understand doubles up to $5 + 5$. This forms the basis of generalising about near doubles. Pupils should also develop an awareness that the sum of any whole number that is doubled will be an even number.

Pupils understand zero can be added to any number but the number will remain unchanged.

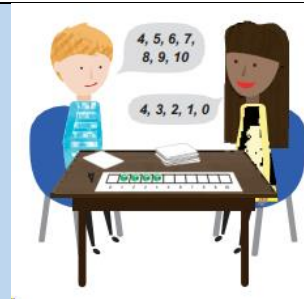
Year 1

Part – Part – Whole



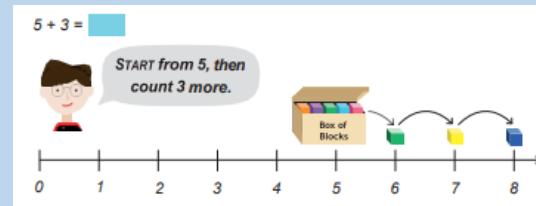
Pupils develop automatic recall of number bonds to 10. This can be shown using a ten frame, a NUMBER bond diagram and written as an equation. This understanding can be related to adding tens, hundreds and so on when used with a sound understanding of place value.

Using a Number Track



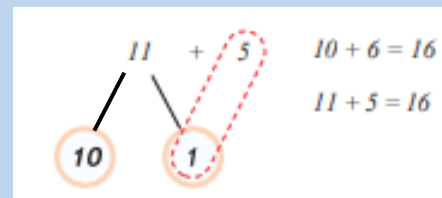
Pupils are first introduced to a linear number system through the number track. This is a precursor to the number line. Pupils may benefit from placing items on the number track as they count and add, before moving on to use the more abstract number line.

Counting on Using a Number Line



Pupils move from a number track to a number line, starting from zero and having marked increments of 1. The use of the number line is further developed when counting starts from a given number, relying on pupils' ability to locate and count on from a given number.

Adding by Making 10



Pupils use their part-whole understanding to rename a number into its component parts in order to make 10 within an equation. Pupils also look for combinations of numbers that make 10 in addition examples that have 3 numbers with a sum greater than 10.

Addition Word Problems

<u>Emma has</u>	<u>Sam has</u>	<u>How many</u>
<u>two balls.</u>	<u>four balls.</u>	<u>balls in</u>
		<u>total?</u>
+	=	

Pupils apply their knowledge of addition within the context of word problems. The problems may involve different situations, contexts or strategies.

Formal Written Method

$43 + 8 =$

START by ADDING the ones.

3 ones + 8 ones = 11 ones
11 ones = 1 ten and 1 one

RENAME 10 ones AS 1 ten.

Then ADD the tens.

tens	ones
4	3
+	
	8
1	1
+	
4	0
5	1

4 tens + 1 ten = 5 tens
 $40 + 10 = 50$
 $43 + 8 = 51$
There ARE 51 bottles of water in total.

Adding Fractions

$\frac{1}{3}$ and $\frac{2}{3}$ make 1 whole.

This is a procedural method that relies on a pupil's conceptual understanding of addition. This begins without renaming and progresses to the renaming of 10 ones into 1 ten. Pupils understand that at this stage, they start with the addition of the ones before they add the tens. This method is supported with base 10 block representation. The formal written method is always accompanied by a written equation to ensure that the relationship between the representations is made.

Pupils use their understanding of adding the same noun when adding fractions through a written sentence. Fractions with the same denominator are added using a '[]' and '[]' make '[]' structure.

Year 3

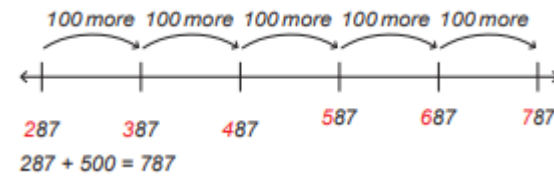
Part – Part – Whole

$9 + 8 = 17$
 $8 + 9 = 17$
 $17 - 9 = 8$
 $17 - 8 = 9$

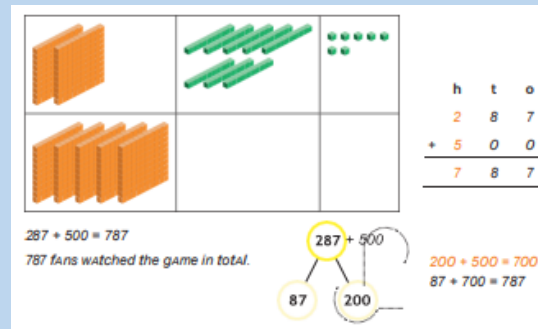
17 is the whole.
8 AND 9 ARE the PARTS.

This is a mathematical structure that underpins all addition situations. numbers can be understood in terms of their parts; understanding that the parts are part of a larger collection. Pupils develop an understanding of the parts and the whole within an equation.

Counting on using a
Number Line



Base 10 Blocks



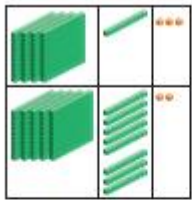
The use of the number line is further developed when counting starts from a given number, relying on pupils' ability to locate and count from a given number, including starting from a 3-digit number. Initially a 1-digit number is added to a 3-digit number, then this progresses to a number line shown with intervals of 1, then 10 and eventually to 100.

The use of base 10 blocks provides a representation of the place value of 3-digit numbers. This representation is related to the formal written method but also encourages pupils to use their understanding of adding the same noun to add 3-digit numbers. For example, $200 + 500$ can be understood as 2 hundreds + 5 hundreds. The sum of these numbers is 700 or 7 hundreds. Progression is made by adding ones, then tens and finally hundreds before the addition of all 3 is undertaken. An understanding of place value will support addition as well as subtraction, multiplication and division.

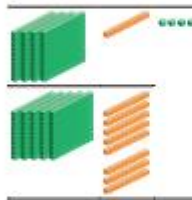
Formal Written Method

$473 + 582 =$

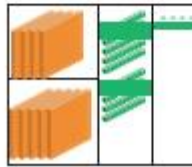
Step 1 Add the ones.
3 ones + 2 ones = 5 ones




Step 2 Add the tens.
7 tens + 8 tens = 9 tens




Step 3 Add the hundreds.
4 hundreds + 5 hundreds = 9 hundreds



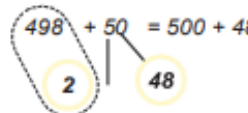

$473 + 582 = 995$

This procedural method progresses from the renaming of 10 ones into 1 ten to include the renaming of 10 tens to 1 hundred. The procedure remains unchanged from Year 2. Pupils understand that at this stage, they start with the addition of the ones, then the tens, then finally the hundreds. This method is supported with base 10 block representation. The formal written method is always accompanied by a written equation to ensure that the relationship between the representations is made.

Adding by making 100

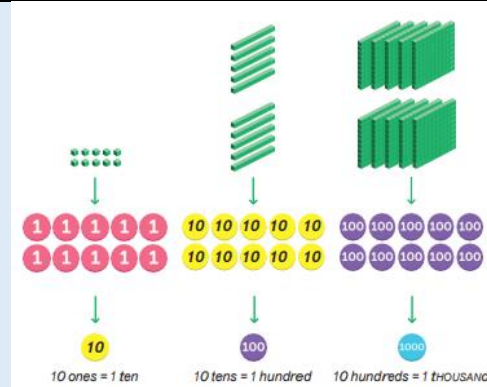
$498 + 50 =$

$498 + 50 = 500 + 48$



Pupils are given the opportunity to further develop their number sense by using a 'make 100' strategy with numbers that are 'near hundreds'. They use their part-whole understanding to rename a given number to make 100. For example, $498 + 50$ can be renamed as $498 + 2 + 48$. Pupils add 2 to 498 to make 500, then add the remaining 48.

Base 10 Blocks



Place-Value Counters

$4506 + 3125 =$

Step 1 Add the ones.
6 ones And 5 ones = 11 ones
Rename the ones.
11 ones = 1 ten And 1 one

4 5 0 6
+ 3 1 2 5

1

Formal Written Method

$4188 + 3245 =$

4	1	8	8				
+	3	2	4	5			
				1	3	Add the ones.	
	1	2	0			Add the tens.	
	3	0	0			Add the hundreds.	
+	7	0	0	0		Add the thousands.	
				7	4	3	3
2	6	1	2				
+	4	2	6	4			
				6	8	7	6

The use of base 10 blocks provides a representation of the place value of 3-digit numbers. This representation is related to the formal written method but also encourages pupils to use their understanding of adding the same noun. In Year 4, a transition between base 10 blocks and place-value counters takes place.

Place-value counters are used to represent addition situations. This transition relies on pupils understanding the value of each counter without being able to count its physical attributes. Pupils will have the opportunity to rename 10 counters of the same value to 1 counter with a value 10 times greater and vice versa. The idea of composing and decomposing at a rate of 10 should be well understood at this stage.

Pupils will have the opportunity to use a long and short version of this procedural method. In the long representation, the sum of adding each place is shown in its entirety before being added to find the final sum. In the short representation, the sum of each place is shown as part of the total sum and as a small number added to an existing place when a ten of

Estimating the Sum

Start by estimating.

$$4188 \approx 4200$$
$$3245 \approx 3200$$
$$4200 + 3200 = 7400$$

The ANSWER will be ABOUT 7400.



Making 10 and making 100

make 10

$$4072 + 8 = \boxed{}$$
$$4072 + 8 = 4070 + 10$$
$$4072 + 8 = 4080$$

make 100

$$97 + 5213 = \boxed{}$$
$$97 + 5213 = 100 + 5210$$
$$= 5310$$

Adding Using Compensation

1 Lulu used this method to find the sum of 3067 And 9.

$$3067 + 10 = 3077$$
$$3067 + 9 = 3076$$

1 less

I know ADDING 9 is 1 less THAN ADDING 10.



2 Ravi used this method to find the sum of 98 And 5262.

$$100 + 5262 = 5362$$
$$98 + 5262 = 5360$$

2 less

I know ADDING 98 is 2 less THAN ADDING 100.

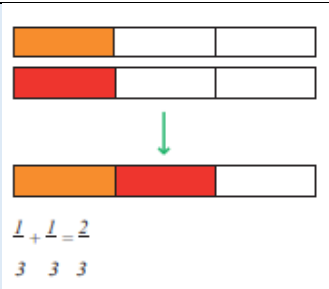
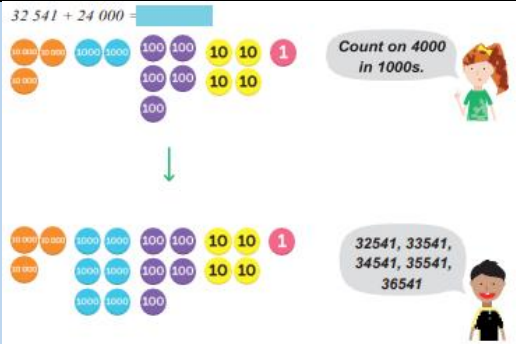
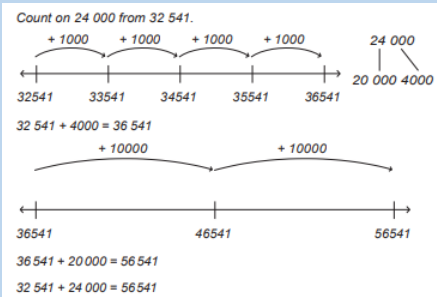
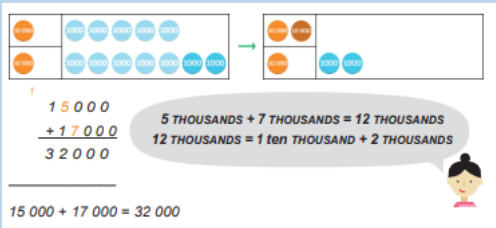


one place is made. The procedure remains unchanged from Year 2.

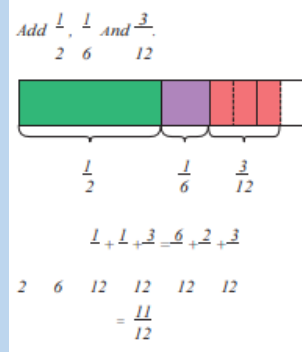
Estimation is introduced as an approach to start a calculation. Estimation is a skill that helps develop number sense. Pupils are expected to be able to decide if an answer is reasonable. Beginning a calculation with estimation is developed during the addition chapter.

A mental method that involves renaming numbers to make 10 or 100 before finding the sum. Pupils develop their number sense by recognising numbers close to a ten or close to a hundred and renaming a number in the equation to bring a number to the nearest 10 or nearest 100 without having to compensate the sum.

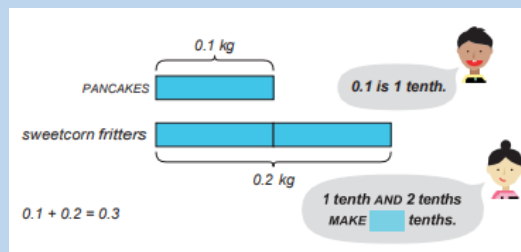
A mental method that uses a similar equation in which a number in the original calculation is shown to the nearest 10 or 100 before carrying out the calculation. This calculation is used to help find the sum of the original equation.

	Adding Fractions		Pupils use their understanding of adding the same noun when adding fractions with the same denominator. The adding of fractions uses equations and is supported through pictorial representation. Pupils use their understanding of equivalence to ensure denominators are the same before carrying out the addition.
Year 5	Counting on Using Place-Value Counters		Pupils use place-value counters to support counting on in thousands to find the sum.
	Counting on Using Number Lines		Pupils count in thousands and ten thousands, using a number line to show this counting on method.
	Formal Written Method		Place-value counters are used to represent the formal written method. The procedure remains unchanged from Year 2.

Adding Fractions



Adding Decimals



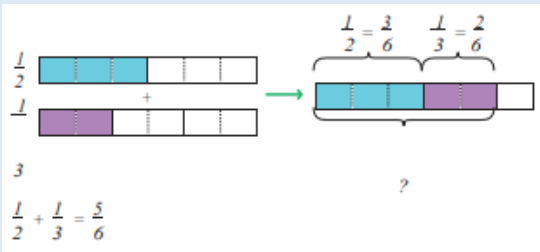
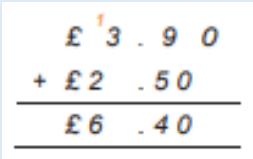
Adding Decimals Using the Formal Written Method

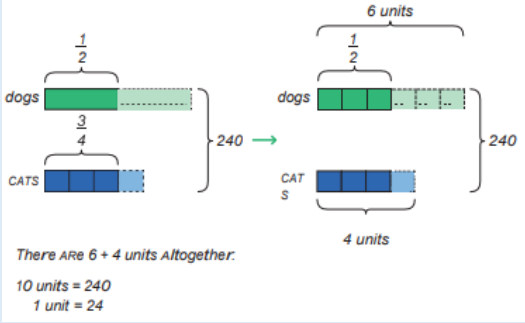
$$\begin{array}{r} £ 1 . 80 \\ + £ 0 . 70 \\ \hline £ 2 . 50 \end{array}$$

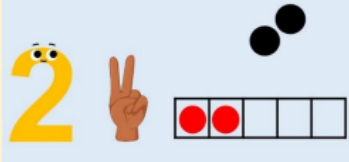
Pupils use their understanding of adding the same noun when adding fractions with the same denominator. The adding of fractions uses equations and is supported through pictorial representation. Pupils use their understanding of equivalence to ensure denominators are the same before carrying out the addition.

Pupils use their understanding of adding the same nouns when adding tenths. Tenths are represented using bar models, written words and equations.

The procedure for adding decimals using a formal written method is the same as when adding whole numbers, but attention needs to be given to the decimal point. The decimal point does not represent a place but separates the whole from the fractional part of a number. Careful alignment is needed when adding decimal numbers using a formal written method.

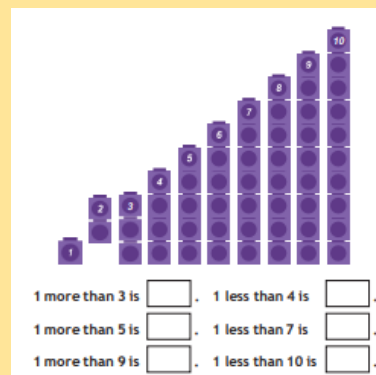
Year 6	Addition within Order of Operations Adding Fractions Adding Decimals	First, carry out All the operations in (). Next, perform All the multiplication And division. Then, calculate All the Addition And subtraction. Calculate. (A) $(1 + 3) \times 5 - 7 =$ (b) $1 + (3 \times 5) - 7 =$ (c) $(1 + 3) \times (7 - 5) =$  $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ 	Pupils utilise the previous addition skills within mixed operation equations. Addition is carried out after multiplication and division. If only addition and subtraction are present in an equation, pupils work from left to right. (BIDMAS) Pupils use their understanding of adding the same noun when adding fractions with the same and different denominators. Pupils use their understanding of equivalence to ensure the nouns and the denominators are the same before the calculation is completed. Pupils use their understanding of adding the same nouns when adding decimal numbers. They use place-value knowledge and composing and decomposing at a rate of 10 when adding decimals. The procedure remains the same as adding whole numbers.
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	Bar Models	 There ARE 6 + 4 units Altogether: 10 units = 240 1 unit = 24		Pupils are expected to utilise previously learned addition skills within increasingly complex situations. The procedure of addition is often at a level previously learned in isolation but the skill being developed is identifying when to use addition within a problem.
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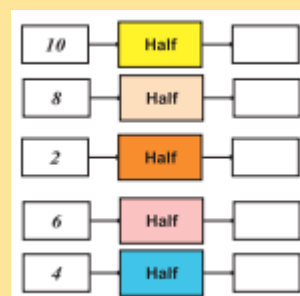
Subtraction				
Year Group	Strand/Topic	Representation		Key Idea
Reception	Perceptual Subitising			A key development underpinning the ability to subitise. Perceptual subitising is when pupils can recognise the quantity of items in groups up to 5 without counting each item.
	Composition			This is a mathematical structure that underpins all subtraction situations. numbers can be understood in a practical /roleplay exploration and through pictorial stories. “How many birds are sitting on the fence?” If one bird flew out of the tree and onto the fence, how many birds can you see in the tree?

Reception	Perceptual Subitising	<table><tr><td></td><td>0</td><td>zero</td></tr><tr><td>•</td><td>1</td><td>one</td></tr><tr><td>• •</td><td>2</td><td>two</td></tr><tr><td>• • •</td><td>3</td><td>three</td></tr><tr><td>•• ••</td><td>4</td><td>four</td></tr><tr><td>•• •• •</td><td>5</td><td>five</td></tr></table>		0	zero	•	1	one	• •	2	two	• • •	3	three	•• ••	4	four	•• •• •	5	five	A key development underpinning the ability to subtract is subitising. Perceptual subitising is when pupils can recognise the quantity of items in groups up to 5 without counting each item.
	0	zero																			
•	1	one																			
• •	2	two																			
• • •	3	three																			
•• ••	4	four																			
•• •• •	5	five																			
	Part-Part-Whole		This is a mathematical structure that underpins subtraction situations. Numbers can be understood in terms of their parts; understanding that the parts are part of a larger collection.																		
	Conceptual Subitising		Pupils are able to recognise different quantities by combining within a group without counting them. Pupils can combine these quantities to find the whole amount. This skill is used when subtracting small amounts.																		
	Composition of 5		Pupils are able to demonstrate all possible whole number compositions of 5, including recognising and showing 5 on a five frame and using a number bond diagram.																		
	Composition of 10		Pupils are able to demonstrate all possible whole number compositions of 10, including recognising and showing 10 on a ten frame and using a number bond diagram.																		

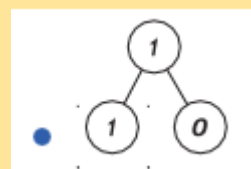
Subtracting 1, 1 Less



Doubles



Subtracting Zero



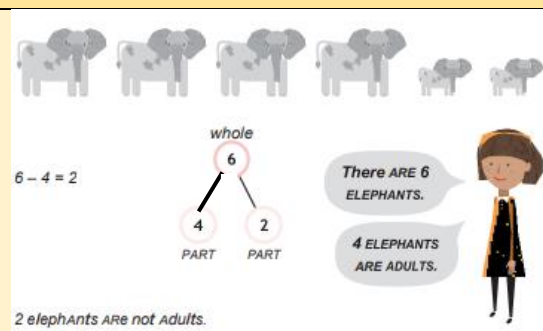
Pupils relate subtracting 1 to one less than the starting number.

By knowing doubles, pupils can find half of a quantity that remains after half the quantity is subtracted.

Pupils understand zero can be subtracted from any number, but the number will remain unchanged.

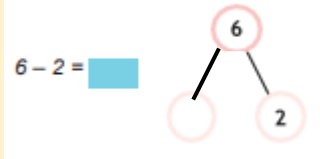
Year 1

Part-Part-Whole



This is a mathematical structure that underpins subtraction situations. Numbers can be understood in terms of their parts; understanding that the parts are part of a larger collection. Pupils develop an understanding of the parts and the whole within an equation.

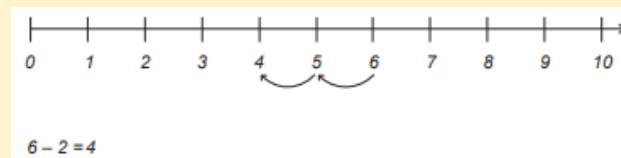
Number Bonds to 10



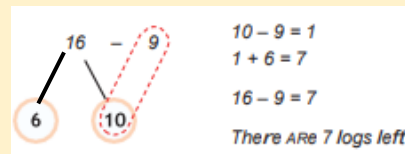
Using a Number Track



Counting Back Using a Number Line



Subtracting from 10



Pupils develop automatic recall of number bonds to 10. This can be shown using a ten frame, a number bond diagram and written as an equation. This understanding can be related to subtracting tens, hundreds and so on when used with a sound understanding of place value.

Pupils are first introduced to a linear number system through the number track. This is a precursor to the number line. Pupils may benefit from placing items on the number track as they count and subtract before moving on to use the more abstract number line.

Pupils move from a number track to a number line, starting from zero and having marked increments of 1. The use of the number line is further developed when counting back starts from a given number, relying on pupils' ability to locate and count back from a given number.

Pupils use their part-whole understanding to rename a number into its component parts in order to subtract from 10 within an equation.

Base 10 Blocks

5 ones - 1 one = 4 ones
 $5 - 1 = 4$

5 tens - 1 ten = 4 tens
 $50 - 10 = 40$

Use tens to help you.

Formal Written Method

tens	ones
5	8
- 4	0
	8

8 ones - 0 ones = 8 ones
 $8 - 0 = 8$

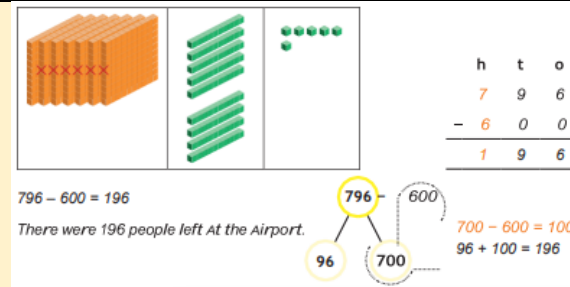
tens	ones
5	8
- 4	0
1	8

5 tens - 4 tens = 1 ten
 $50 - 40 = 10$
 $58 - 40 = 18$

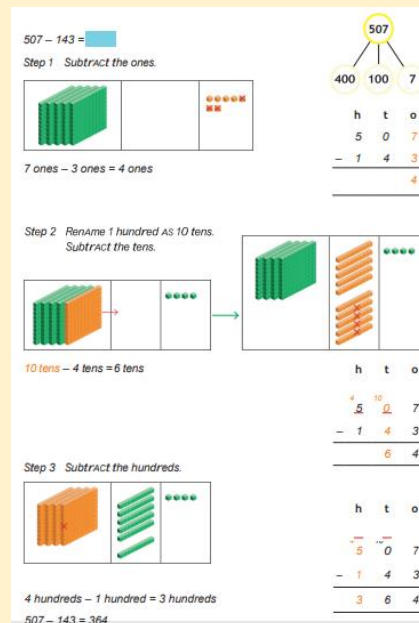
The use of base 10 blocks provides a representation of the place value primarily of 2-digit numbers. This representation is related to the formal written method but also encourages pupils to use their understanding of subtracting the same noun to subtract 2-digit numbers. For example, $50 - 30$ can be understood as 5 tens - 3 tens. The difference between the numbers is 20 or 2 tens. An understanding of place value will support subtraction as well as addition, multiplication and division.

This is a procedural method that relies on a pupil's conceptual understanding of subtraction. Initially, this begins without renaming and progresses to the renaming of 1 ten into 10 ones. Pupils understand that at this stage, they start with the subtraction of the ones before they subtract the tens. This method is supported with base 10 block representation. The formal written method is always accompanied by a written equation to ensure that the relationship between the representations are made.

Base 10 Blocks



Formal Written Method



The use of base 10 blocks provides a representation of the place value of 3-digit numbers. This representation is related to the formal written method but also encourages pupils to use their understanding of subtracting the same noun to subtract from 3-digit numbers. For example, 700 - 400 can be understood as 7 hundreds - 4 hundreds. The difference between these numbers is 300 or 3 hundreds. Progression is made by subtracting ones, then tens and finally hundreds before the subtraction of all 3 places is undertaken. An understanding of place value will support subtraction as well as addition, multiplication and division.

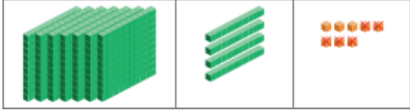
This procedural method progresses from the renaming of 10 ones into 1 ten to include the renaming of 10 tens to 1 hundred when necessary. The procedure itself remains unchanged from Year 2. Pupils understand that at this stage, they start with the subtraction of the ones, then the tens, then finally the hundreds. This method is supported with base 10 block representation. The formal written method is always accompanied by a written equation to ensure that the relationship

Inverse Operation

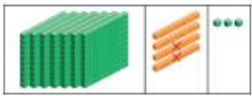
Difference Using a Bar Model

$748 - 425 =$

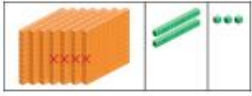
Step 1 Subtract the ones.
8 ones - 5 ones = 3 ones



h	t	o
7	4	8
-	4	2
		3



Step 3 Subtract the hundreds.
7 hundreds - 4 hundreds = 3 hundreds



h	t	o
7	4	8
-	4	2
		3

$748 - 425 = 323$
323 tomatoes are left.

Check your ANSWER using ADDITION.
 $323 + 425 = 748$

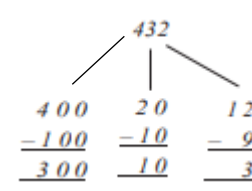
between the representations are made.

Pupils should understand that subtraction is the inverse operation of addition. They are encouraged to check completed subtraction calculations using addition.

Pupils are required to find the difference in a comparison problem when represented by a bar model. To find the difference, the known part is subtracted from the quantity it is being compared to. The comparison model reinforces the understanding of difference in subtraction.

Year 4

Part-Part-Whole



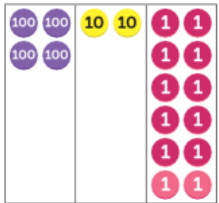
This is a mathematical structure that underpins subtraction situations. Numbers can be understood in terms of their parts; understanding that the parts are part of a larger collection. Pupils develop an understanding of the parts and the whole within an equation.

Place-Value Counters

What is the difference between 432 and 119?

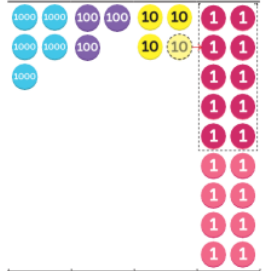


There ARE not enough ones.
RENAME 1 ten AS 10 ones.

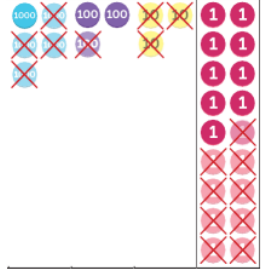


$$\begin{array}{r} 400 \\ - 100 \\ \hline 300 \end{array} \quad \begin{array}{r} 20 \\ - 10 \\ \hline 10 \end{array} \quad \begin{array}{r} 12 \\ - 9 \\ \hline 3 \end{array}$$

Formal Written Method



RENAME 1 ten to 10 ones.

$$\begin{array}{r} 5318 \\ - 4139 \\ \hline \end{array}$$


Now there ARE enough ones to SUBTRACT.

$$\begin{array}{r} 5318 \\ - 4139 \\ \hline 1209 \end{array}$$

Place-value counters are used to represent subtraction situations. This transition from base 10 blocks relies on pupils understanding the value of each counter without being able to count its physical attributes. Pupils will have the opportunity to rename 1 counter to 10 counters with a value 10 times smaller in order to carry out a formal written method. The idea of decomposing at a rate of 10 should be well understood at this stage.

Pupils will use the formal written method initially without renaming, and then move to subtraction that requires renaming. The procedure remains the same as learned in Year 3 but the numbers increase to include 4-digit numbers being subtracted from 4-digit numbers.

Using Addition to Check Subtraction

5348

5000 300 30 18

Step 1 Subtract the ones.
18 ones – 9 ones = 9 ones

Step 2 Subtract the tens.
3 tens – 3 tens = 0 tens

Step 3 Subtract the hundreds.
3 hundreds – 1 hundred = 2 hundreds

Step 4 Subtract the thousands.
5 thousands – 4 thousands = 1 thousand

5348 – 4139 = 1209

Check.
1 2 0 9
+ 4 1 3 9
5 3 4 8



Mental Methods

3002 – 198 = 2804

3002
2802 200

3002 – 198 = 2802 + 2

200 – 198 = 2



Subtracting Fractions

$3 - \frac{7}{10} = 2\frac{10}{10} - \frac{7}{10} = 2\frac{3}{10}$

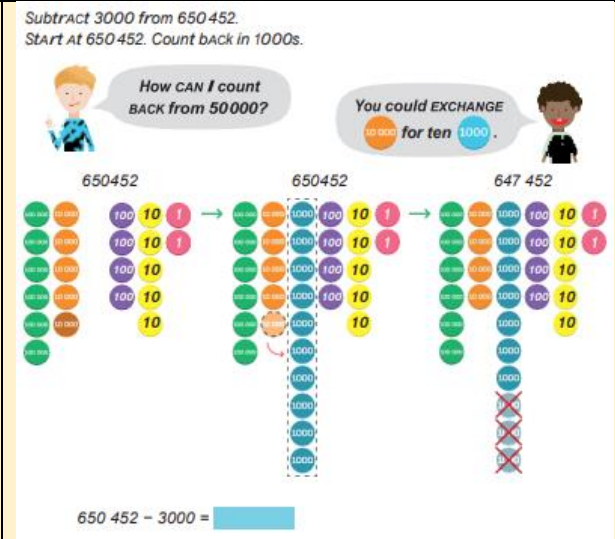
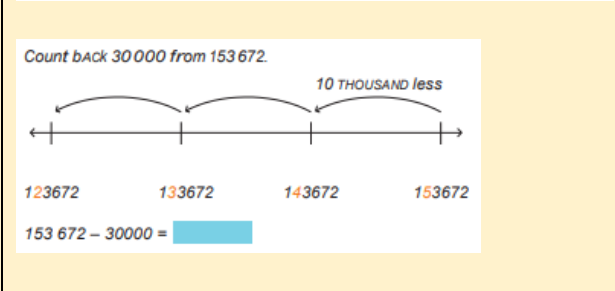
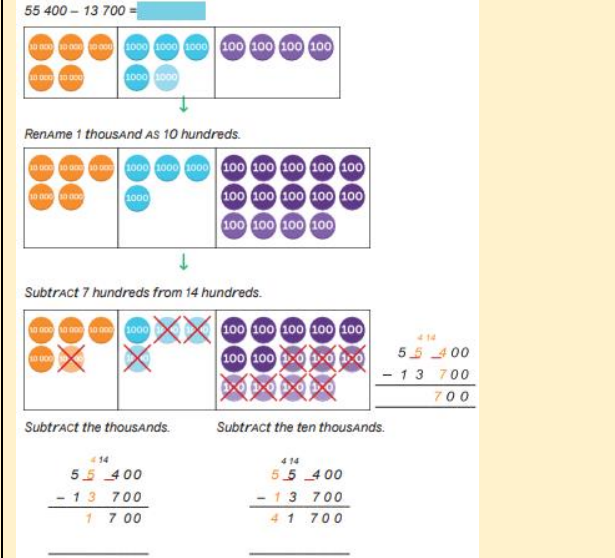
$2\frac{10}{10} - \frac{7}{10} = 2\frac{3}{10}$



Pupils are encouraged to check subtraction calculations by adding the parts (the subtrahend and the difference) to ensure the sum is equal to the whole (the minuend).

Mental subtraction methods include partitioning the minuend to simplify the subtraction calculation. The approach shown is supported by an understanding of number bonds to 10 and to 100.

Pupils use their understanding of subtracting the same nouns when subtracting fractions with the same denominator. The subtraction of fractions or finding the difference between fractions is supported through pictorial representation. Pupils use their understanding of equivalence to ensure denominators are the same before carrying out the subtractions.

Year 5	Counting Back Using Place-Value Counters	Subtract 3000 from 650452. Start At 650452. Count back in 1000s.  How CAN I count BACK from 50000? You could EXCHANGE 10,000 for ten 1,000. 650452 650452 647452 650 452 - 3000 =	Pupils use place-value counters to support counting back in thousands to find the difference.
	Counting Back Using Number Lines	Count back 30 000 from 153 672.  10 THOUSAND less 123672 133672 143672 153672 153 672 - 30000 =	Pupils count back in thousands and ten thousands, using a number line to show this counting back method.
	Formal Written Method	55 400 - 13 700 =  Rename 1 thousand AS 10 hundreds. Subtract 7 hundreds from 14 hundreds. Subtract the thousands. Subtract the ten thousands. 55 400 - 13 700 = 41 700	Place-value counters are used to represent the formal written method. The procedure to subtract using numbers up to 6-digits using the formal written method remains the same as when it was first introduced. Pupils begin at the least value place and work to the left through the places to find the difference. Renaming takes place when a calculation in a place cannot be done. Again, this procedure is the same as when this was first learned and requires the renaming of the

Checking Using
Estimation and Addition

$75\,241 - 34\,658 = 40\,583$

$$\begin{array}{r} 4\,0\,5\,8\,3 \\ + 3\,4\,6\,5\,8 \\ \hline 7\,5\,2\,4\,1 \end{array}$$

I checked my ANSWER using ADDITION.

$75\,241 - 34\,658 \approx 75\,000 - 35\,000 = 40\,000$

I checked my ANSWER using ESTIMATION.

Subtracting Fractions

$1 - \frac{1}{6} = \frac{6}{6} - \frac{1}{6} = \frac{5}{6}$

$\frac{5}{6} - \frac{5}{12} = \frac{10}{12} - \frac{5}{12} = \frac{5}{12}$

$\frac{5}{6} = \frac{10}{12}$

Subtracting Decimals

Find the difference between 0.7 kg And 0.3 kg.

0.7 kg

0.3 kg

?

$0.7 - 0.3 = 0.4$

minuend. The renaming of the minuend is also represented using a number bond, providing the foundation for mental methods that require renaming.

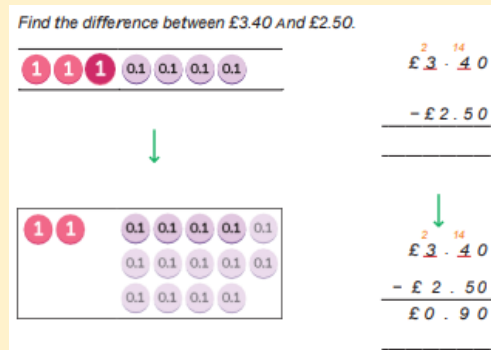
Pupils are encouraged to check the reasonableness of their answers by initially finding an estimated difference. When using estimation to check, pupils initially round to the nearest thousand before calculation. When using addition to check the difference, pupils add the difference and the subtrahend to check it is equal to the minuend.

Pupils use their understanding of subtracting the same nouns when subtracting fractions with the same denominator. The subtraction of fractions or finding the difference between fractions is supported through pictorial representation. Pupils use their understanding of equivalence to ensure denominators are the same before carrying out the subtractions.

Pupils use their understanding of subtracting the same nouns when subtracting tenths. Tenths are represented using bar models, written words and equations.

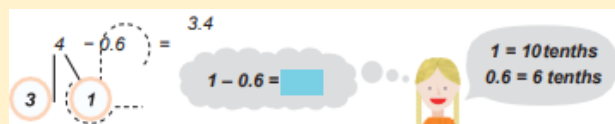
Subtracting Decimals
Using the Formal Written
Method

Find the difference between £3.40 and £2.50.



The image shows base ten blocks representing 3.40 (three 1-unit blocks, four 0.1-unit blocks) and the formal written method for subtracting 2.50 from 3.40. The formal method shows the numbers aligned by their decimal points, with a 2 being carried from the tenths column to the units column to facilitate subtraction.

Subtracting Decimals
Using Equivalence



The diagram shows a number line for 4 - 0.6. A box labeled '1' is shown, with a speech bubble stating '1 = 10 tenths' and '0.6 = 6 tenths'. The calculation 4 - 0.6 = 3.4 is shown, with a thought bubble containing '1 - 0.6 = 0.4'.

The same procedure for subtracting decimals using a formal written method is the same as when subtracting whole numbers but attention needs to be given to the decimal point. The decimal point does not represent a place but separates the whole from the fractional part of a number. Careful alignment is needed when subtracting decimal numbers using a formal written method.

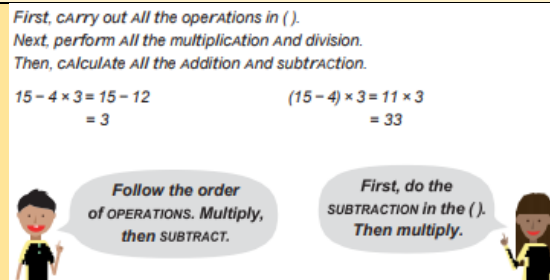
Pupils use their understanding of equivalence to subtract a decimal from a whole number. For example, when calculating $4 - 0.6$ we can rename 4 as 40 tenths, so the calculation becomes 40 tenths - 6 tenths. Once the nouns are the same, the subtraction can be carried out. 40 tenths - 6 tenths = 34 tenths = 3.4

Year 6

Subtraction within Order
of Operations

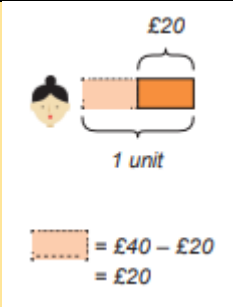
First, carry out All the operations in ().
Next, perform All the multiplication And division.
Then, calculate All the Addition And subtraction.

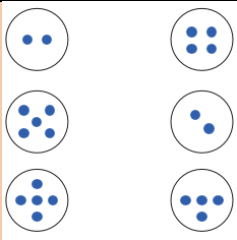
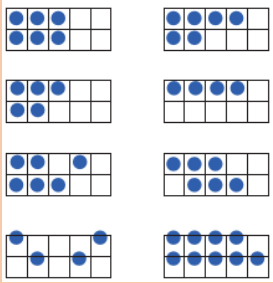
$$15 - 4 \times 3 = 15 - 12 = 3$$

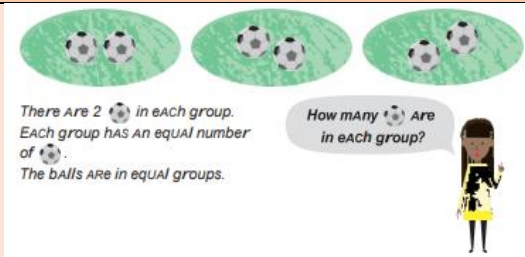
$$(15 - 4) \times 3 = 11 \times 3 = 33$$


The diagram shows two examples of the order of operations. The first example shows the calculation $15 - 4 \times 3 = 15 - 12 = 3$, with a speech bubble stating 'Follow the order of OPERATIONS. Multiply, then SUBTRACT.' The second example shows the calculation $(15 - 4) \times 3 = 11 \times 3 = 33$, with a speech bubble stating 'First, do the SUBTRACTION in the (). Then multiply.'

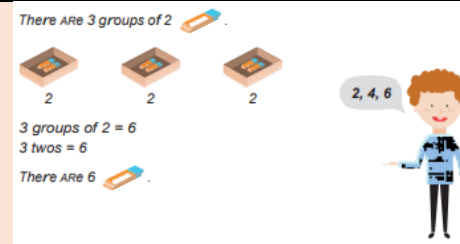
Pupils utilise the previous subtraction skills within mixed operation equations. Subtraction is carried out after multiplication and division. If only addition and subtraction are present in an equation, pupils work from left to right. (BIDMAS)

	Bar Models		Pupils are expected to utilise previously learned subtraction skills within increasingly complex situations. The procedure of subtraction is often at a level previously learned in isolation but the skill being developed is identifying when to use subtraction within a problem.
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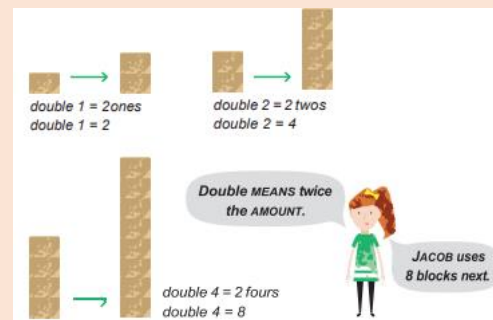
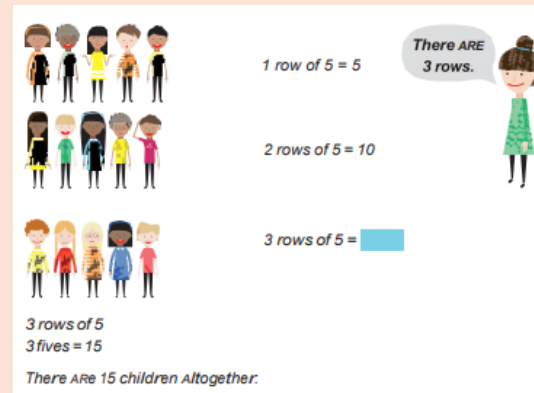
Multiplication			
Year Group	Strand/Topic	Representation	Key Idea
Reception	Equal Groups		Pupils learn to recognise groups that are equal in quantity, initially using like items and then progressing to different items. Pupils understand that equal groups can be represented by concrete items, diagrams and written numbers. Pupils need to be secure in the abstraction principle of counting the quantity of items, regardless of the properties or characteristics of the items, in order to recognise equal groups in a range of situations.
	Addition		

			equal groups when doubling numbers.
Year 1	Equal Groups  Repeated Addition  Counting in 2s, 5s and 10s	Pupils learn to recognise groups that are equal in quantity, initially using like items and then progressing to different items. Pupils understand that equal groups can be represented by concrete items, diagrams and written numbers. Pupils need to be secure in the abstraction principle of counting the quantity of items, regardless of the properties or characteristics of the items, in order to recognise equal groups in a range of situations. Initially, multiplication is shown as the addition of equal groups. The key idea of adding like nouns still applies in multiplication. A group of 3 bananas and 3 apples does not result in 6 bananas or 6 apples. In order to add, the nouns must be the same, in this case 6 pieces of fruit. This is also true of multiplication: 2 groups of 3 pieces of fruit makes 6 pieces of fruit. Pupils start to count in multiples of 2 and multiples of 10, then progress to counting in multiples of 2, 5 and 10	

Arrays



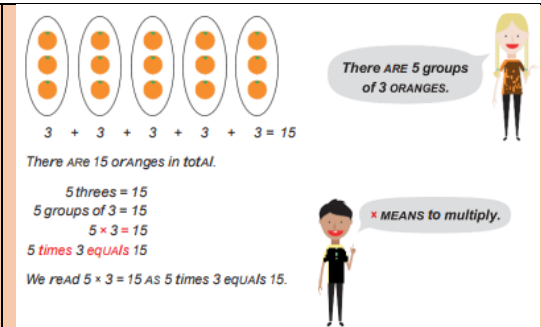
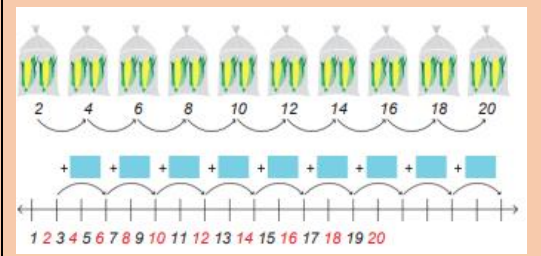
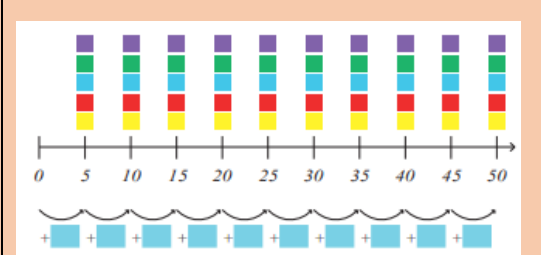
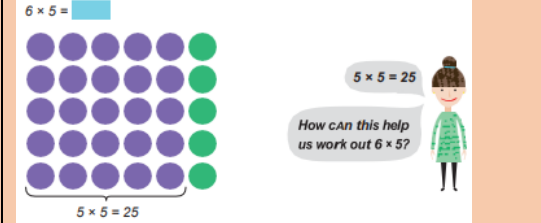
Doubles

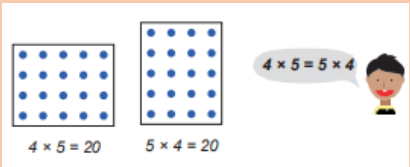
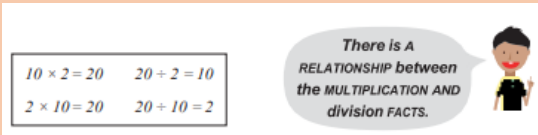
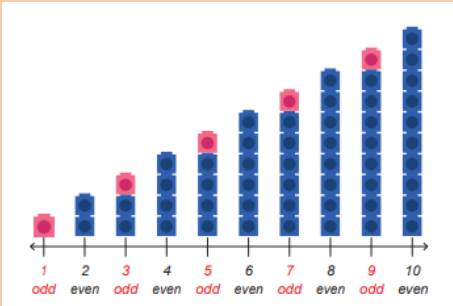
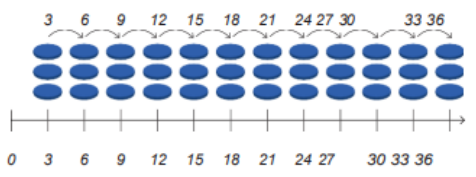


supported by discrete, countable representations.

Multiplication is represented by arrays, beginning with making equal rows and further developing the language associated with arrays. For example: 'There are 3 rows of 5. There are 15 altogether.'

The diagrams used to support learning how to double numbers, not only show equal groups of 2 being added each time, but also show the pattern scaling up and each 'tower' being twice the height of the tower just before it. Pupils can develop the language associated with multiplication by describing the growing block pattern. This also provides the basis for understanding halving, in which the representation scales down.

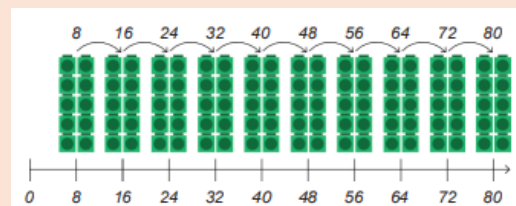
Year 2	Equal Groups			Pupils learn to recognise groups that are equal in quantity, initially using like items and then progressing to different items. Pupils understand that equal groups can be represented by concrete items, diagrams and written numbers. In Year 2, the progression to multiplication from repeated addition is shown as $3 + 3 + 3 + 3 + 3$ being equal to 5 groups of 3 and 5 groups of 3 being equal to 5×3. Pupils read 5×3 as 5 groups of 3.
	Counting in 2s, 5s and 10s			When a pupil knows that the size of a group is 2, 5 or 10 and the group size remains consistent, they can count in multiples of 2, 5 and 10 to find the product. Counting in multiples is supported by representation on a number line.
	Number Line			Counting in multiples is shown on a number line. The increasingly abstract nature of the number line is shown as intervals change from 1 to 2, 5 and 10.
	Associated Facts			As pupils become more fluent and their understanding of their times tables increases, they are expected to use this knowledge to calculate associated facts. A pupil should be

	Commutativity Fact Families Odd and Even Numbers	  	able to relate 10×5 to 9×5, knowing that the latter expression is 1 group of 5 less. So, $9 \times 5 = 50 - 5$. Pupils learn that the order of the factors in an equation does not affect the product. This is supported pictorially through the use of arrays. Pupils relate multiplication and division and see the connection between them when completing fact families. Pupils develop an understanding that $\text{factor} \times \text{factor} = \text{product}$ and $\text{product} \div \text{factor} = \text{factor}$. Once the understanding of this is secure, pupils can relate this to both multiplication and division situations. Pupils develop an understanding that even numbers can be put into groups of 2 exactly but when odd numbers are grouped in twos, there is always 1 remaining.
Year 3	Counting in 3s, 4s and 8s		When a pupil knows that the size of a group is 3, 4 and 8 and the group size remains consistent, they can count in multiples of 3, 4 and 8 to find the product. Counting in multiples is supported by representation on a number line.

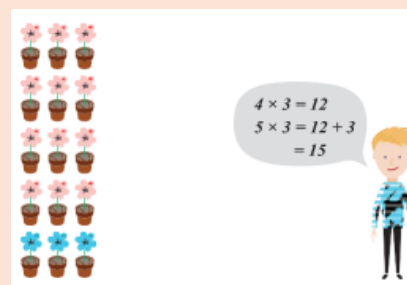
Equal Groups



Number Line



Associated Facts



Number Patterns



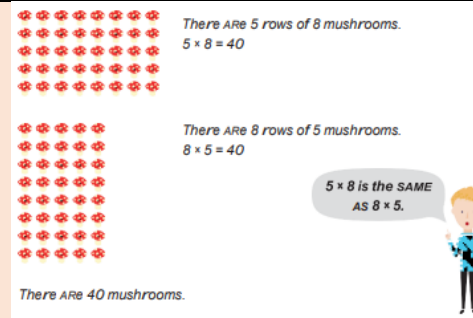
Multiplication by 3, 4 and 8 is shown initially using equal groups. Specific language is used to support these examples, in this case '4 groups of 3', and this is immediately followed by the equation 4×3 . This forms the basis of using known facts to find unknown facts.

Counting in multiples is shown on a number line. Multiples of 3, 4 and 8 are used as the intervals on a number line to support skip counting using these multiples.

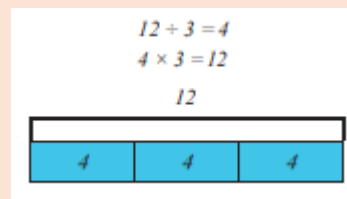
Once the understanding of multiplication as the adding of equal groups is secure, this knowledge can be used to find unknown facts. For example, if a pupil knows 5×3 as 5 groups of 3, they can understand that 6×3 is simply 1 more group of 3. So, $6 \times 3 = 15 + 3$; 4×3 is seen as 1 group fewer than 5×3 ; $4 \times 3 = 15 - 3$. This structure is used in all multiplication tables.

Pupils count in multiples of 3, 4 or 8 to identify missing multiples in a sequence. This reinforces the products found within the 3, 4 and 8 times tables.

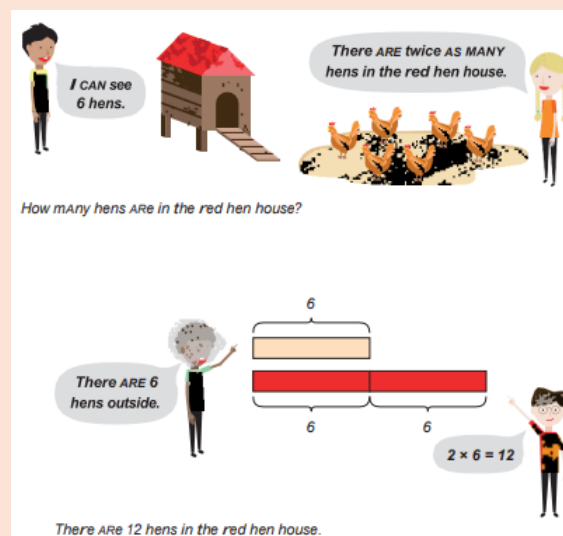
Commutativity



Fact Families



Multiplication Using Bar Models

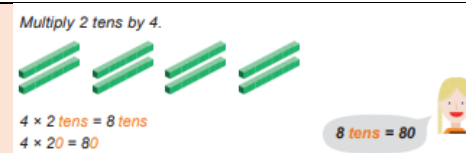


The representation of multiplication as an array is used to further develop the understanding of commutativity. Having first understood multiplication as [] groups of [], pupils develop an understanding that 5×3 can also be read as 5 multiplied 3 times. Pupils should have a firm understanding that the order the factors are multiplied in does not change the product.

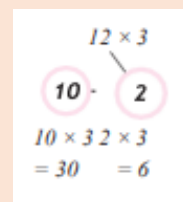
The relationship between multiplication and division is shown using fact families. The product is a result of multiplying factors and dividing the product by a factor will equal the factor used during multiplication.

Bar models are used in multiplicative comparison problems. Pupils use multiplication skills to determine quantities in comparison to another quantity. Language such as 'twice as many', 'three times as many' and so on is developed in relation to multiplicative comparison problems.

Base 10 Blocks



Number Bonds



Written Formal Method

Step 1 Multiply the ones.

$6 \text{ ones} \times 4 = 24 \text{ ones}$
 $24 \text{ ones} = 2 \text{ tens} + 4 \text{ ones}$

Step 2 Multiply the tens.

$3 \text{ tens} \times 4 = 12 \text{ tens}$
 $12 \text{ tens} + 2 \text{ tens} = 14 \text{ tens}$

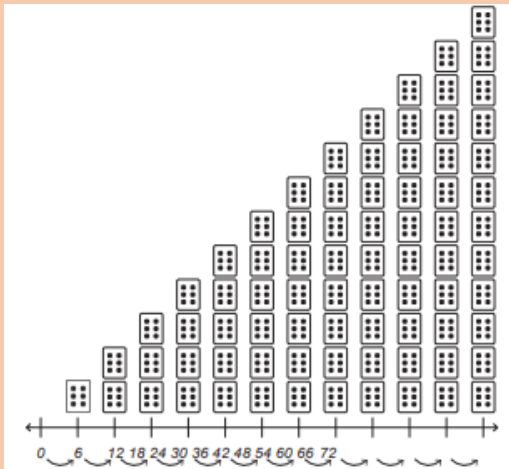
$36 \times 4 = 144$

	2 tens	3	6
	x	4	
		4	4 ones
		12	
		12	
		14	

Base 10 blocks are used to support the understanding of multiplication of 2-digit numbers. Language and understanding is developed through the representation of 3×20 as $3 \times 2 \text{ tens} = 6 \text{ tens}$. Pupils use known multiplication tables to 10 together with the place-value names of the digits being used to carry out the multiplication.

Number bonds are used to show numbers partitioned into tens and ones before being multiplied. The examples being used move from a number bond relating to an equation to an equation and the formal written method.

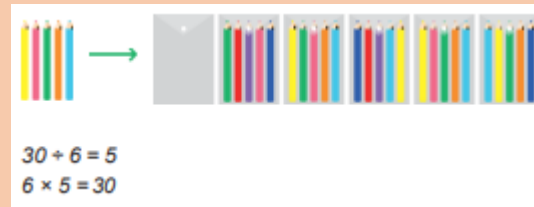
This method is used to multiply a 2-digit number by a 1-digit number. Initially, the method shows the product of the multiplication of the ones, then the product of the multiplication of the tens, before adding the products to find the total. This method progresses to include renaming and finally moves to a shortened form of the written method. The method is finally shown as a version of the formal written method, in which the product of the multiplication of each place is shown as a single product, with any

			renaming added above each place in the multiplication.																														
Year 4	Counting in 6s, 7s, and 9s Equal Groups Number Line	Count on in sixes. <table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr><tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr></table>  	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	When pupils know that the size of a group is 6, 7 and 9 and the group size remains consistent, they can count in multiples of 6, 7 and 9 to find the product. Counting in multiples is supported by representation on a number line using intervals of 6, 7 and 9. Multiplication by 6, 7 and 9 is shown initially using equal groups. Specific language is used to support these examples, in this case ‘4 groups of 6’, and this is immediately followed by the equation 4×6. This forms the basis of using known facts to find unknown facts. Counting in multiples is shown on a number line. Multiples of 6, 7 and 9 are used as the intervals on a number line to support skip counting using these multiples. A growing pattern in multiples of 6, 7 and 9 is also shown to support pupils’ understanding.
1	2	3	4	5	6	7	8	9	10																								
11	12	13	14	15	16	17	18	19	20																								
21	22	23	24	25	26	27	28	29	30																								

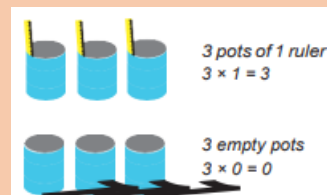
Multiplying by 11 and 12
Using Associated Facts



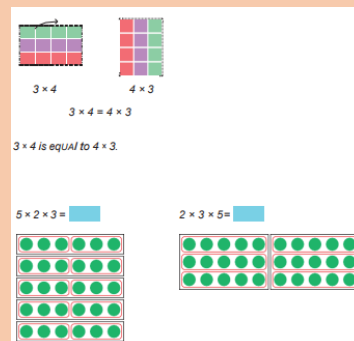
Fact Families



Multiplying by 0 and 1



Commutativity



Learning to multiply by 11 and 12 is supported by partitioning 11 and 12 and using the 10 TIMES table as the basis for initial understanding, building towards immediate recall.

Fact families are used in the introduction of division, represented using arrays to show the relationship between factors and a product. Pupils relate $6 \times 11 = 66$ to $66 \div 6 = 11$. They understand that multiplication can be used in division calculations.

Pupils initially use their understanding of 'groups of' to understand multiplying by zero. For example, 0×4 is read as 'There are zero groups of 4'. Pupils' understanding then moves to read 0×4 as zero multiplied 4 times. The language is an extension of what they have already learned about multiplication.

Arrays are used to support the understanding of commutativity. Pupils learn the pattern of $a \times b = b \times a$. Regardless of the order in which the factors are multiplied, the product remains the same. The commutative property is further developed through the

Multiplying Multiples of 10

30 is equal to 3 tens.

$$5 \times 3 = 15$$

$$5 \times 3 \text{ tens} = 15 \text{ tens} = 150$$


$$5 \times 30 = 150$$

Formal Written Method

2 1 8		
x 4		

3 2	→	$8 \times 4 = 32$
4 0	→	$10 \times 4 = 40$
+ 8 0 0	→	$200 \times 4 = 800$

8 7 2	→	$218 \times 4 = 872$

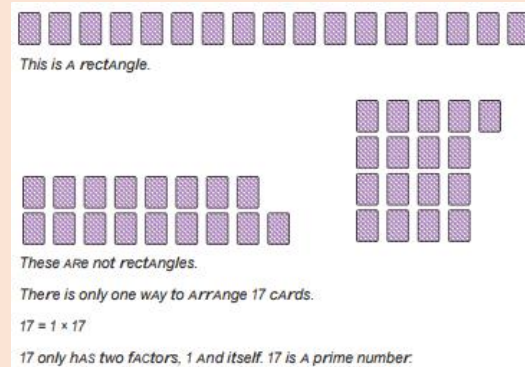
multiplication of 3 numbers. 3 factors are multiplied in different orders and the product remains the same.

Pupils learn to scale a product by a factor of 10 when multiplying a multiple of 10. For example, we know $3 \times 4 = 12$, therefore the product of 30×4 is 10 times greater: $30 \times 4 = 120$. Naming the place value of the digit supports this approach and pupils relate a known fact to multiplying multiples of 10. For example, we can read 30×4 as 3 tens $\times$ 4. So, 3 tens $\times$ 4 = 12 tens or 120. We would expect pupils to generalise and see that $30 \times 4 = 3 \times 4 \times 10$. While this isn't formalised, this forms the basis of the distributive property of multiplication.

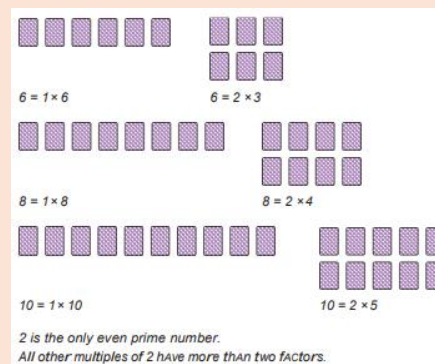
Pupils use formal written methods, short and long, to multiply a 2-digit number by a 1-digit number. Initially the long method is used, showing the product of the multiplication of the ones, tens and hundreds, before adding the products to find the total. Pupils are shown the corresponding short formal written method so can make the links between the two procedures. Multiplication then moves from a 2-digit number by a 1-digit number to a 3-digit number by

			a 1-digit number. Pupils should be aware that even though the number of digits in one number increases, the procedure remains the same.
Year 5	Multiples Finding Factors	 1 row of 8 stamps. $1 \times 8 = 8$  2 rows of 8 stamps. $2 \times 8 = 16$  3 rows of 8 stamps. $3 \times 8 = 24$  4 rows of 8 stamps. $4 \times 8 = 32$ A multiple is A number you get when you multiply one number by ANOTHER number. 8, 16, 24, 32 AND 40 ARE multiples of 8.  5 rows of 8 stamps. $5 \times 8 = 40$ The product of 5 AND 8 is 40. 40 is A multiple of 5. 40 is ALSO A multiple of 8. Sam has 40 stamps Altogether.  2 rows of 12 tiles $2 \times 12 = 24$ 2 AND 12 ARE FACTORS of 24. FACTORS ARE the numbers we multiply together to MAKE ANOTHER number. 2 AND 12 ARE FACTORS of 24 BECAUSE $2 \times 12 = 24$.	Finding multiples is initially related to skip counting. Pupils develop an understanding that counting in 2s produces a series of multiples that are also a product when 2 is a factor. They develop an understanding that the product is the multiple of two numbers. Pupils have already been working with factors for a significant amount of time but the term ‘factors’ is introduced in Year 5. The structure for introducing factors uses rectangular arrangements and identifies the number of rows and number of items in each row. Pupils’ understanding of factors is further developed when looking at common factors. They learn that different numbers can share some of the

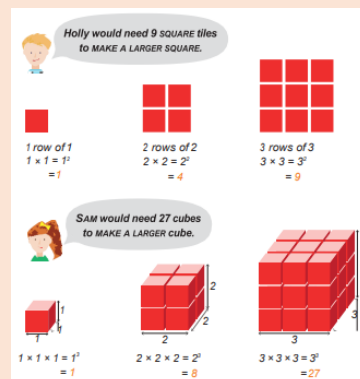
Prime Numbers



Composite Numbers



Square and Cube Numbers



same factors. Pupils may go on to generalise about common factors. For example, all integers that end in 0 or 5 have 5 as a common factor.

Following on from finding factors, pupils use rectangular arrangements to identify a pattern presented by prime numbers. Pupils find that prime numbers can only be arranged in a single rectangular pattern. This leads them to see that certain numbers only have two factors. These numbers, integers greater than 1, are called prime numbers.

Once pupils have a sound understanding of multiples, factors and prime numbers, the term 'composite numbers' is used to describe integers, greater than 1, that have more than two factors.

Pupils are introduced to both square and cube numbers by the physical representation described by their names. These representations lead to abstraction, with pupils understanding that square numbers are the product of a number multiplied by itself and a cube

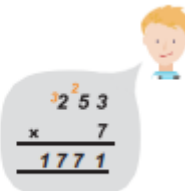
Multiplying by 10, 100 and 1,000

$$5 \times 1000 = \text{[blue box]}$$
$$5 \times 1 \text{ thousand} = 5 \text{ thousands}$$
$$5 \times 1000 = 5000$$

Written formal Method

Multiply 253 by 17.

	2	5	3	
x		1	7	
	1	7	7	1
+	2	5	3	0
	4	3	0	1

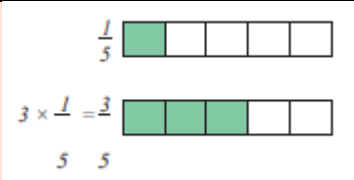
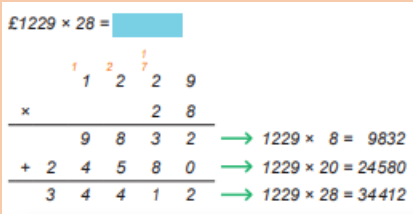


2 5 3
x 7
1 7 7 1

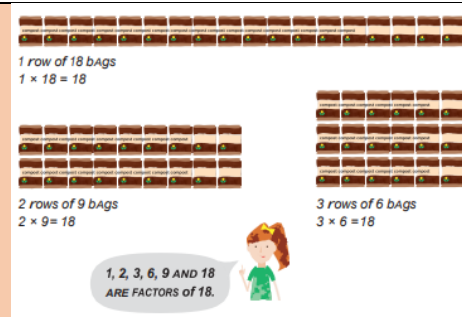
number is the product made by multiplying a number twice by itself.

Pupils build on their understanding of multiplication by factors of 10. They see that when a factor is made 10 times greater, the product is 10 times greater. Pupils use their knowledge of times tables to underpin multiplying by 10, 100 and 1000, so 5×1000 is equal to $5 \times 1 \text{ thousand} = 5 \text{ thousands}$ or 5000. This follows a pattern that has been introduced in previous years.

Pupils use formal written methods, short and long, to multiply a 3-digit number by a 1-digit number; then move on to multiplying a 4-digit number by a 1-digit number. Initially the long method is used, showing the product as a result of multiplying each place. Pupils then progress to the short formal written method making a link between the two procedures. Next, pupils learn to multiply a 2-digit number by a 2-digit number, then a 3-digit number by a 2-digit number. Links are made to the formal written procedure that they know. Pupils work systematically through the procedure progressing from multiplying by ones to multiplying by tens and ones.

	Multiplying Fractions		Multiplying a fraction by a whole number is underpinned by the early idea of adding equal groups. Pupils understand that we need to add and multiply items that have the same noun. We read $\frac{1}{5} \times 3$ as 1 fifth $\times$ 3 = 3 fifths, in the same way we would read 1 kg $\times$ 3 = 3 kg. Bar models are used as pictorial support to show the multiplication of fractions with the same denominator. Pupils progress to multiplying mixed numbers by whole numbers. The approach remains the same but uses partitioning, so pupils multiply the fraction and whole number separately and add the products.
Year 6	Order of Operations	First, carry out All the operations in (). Next, perform All the multiplication And division. Then, calculate All the Addition And subtraction. $15 - 4 \times 3 = 15 - 12 = 3$ $(15 - 4) \times 3 = 11 \times 3 = 33$ 	Pupils use the multiplication skills they have learned in previous years within expressions and equations that use multiple operations. Pupils learn to multiply within brackets first, then left to right in expressions and equations that use multiplication. The procedures to multiply remain the same throughout.
	Multiplying by 2-Digit Numbers		Pupils revisit the formal written method, multiplying up to 4-digit numbers by 2-digit numbers.

Common Factors

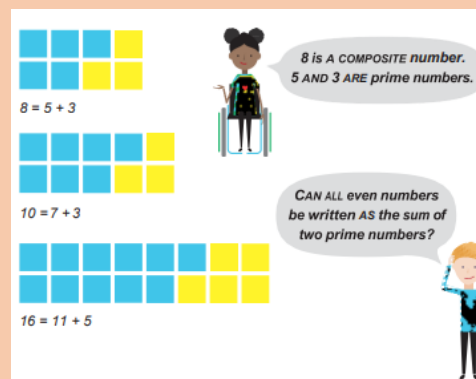


Common Multiples

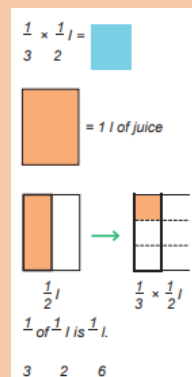
Multiples of 4	4	8	12	16	20	24	28	32	36	40	44	48
Multiples of 6	6	12	18	24	30	36	42	48	54	60	66	72
Multiples of 8	8	16	24	32	40	48	56	64	72	80	88	96

24 And 48 ARE common multiples of 4, 6 And 8.

Prime Numbers



Multiplying Fractions

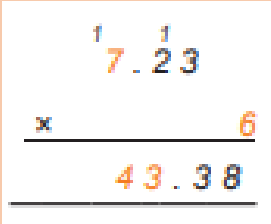


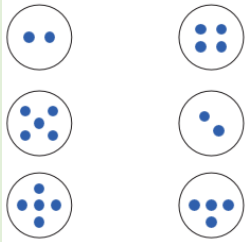
Prior learning is expanded on by finding common factors within more challenging word problems. Pupils are encouraged to partition larger numbers into known multiples to determine if the given number is a factor.

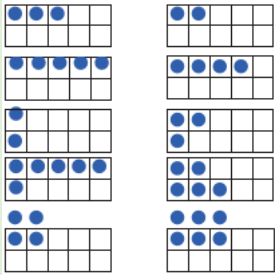
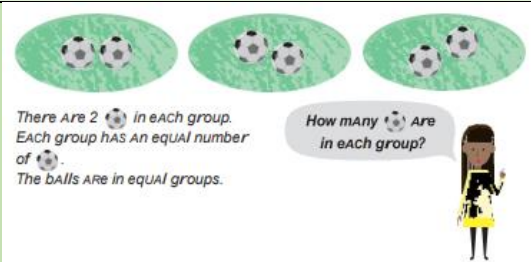
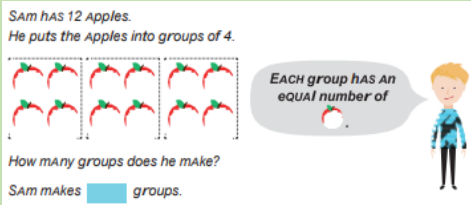
Pupils are introduced to common multiples with the understanding that they are a multiple of 2 or more numbers.

Pupils' understanding of prime numbers is expanded through the use of Goldbach's conjecture, that all even numbers greater than 2 can be expressed as the sum of two prime numbers.

Pupils learn to multiply proper fractions by proper fractions. They read fractions to support multiplication, so $\frac{1}{3} \times \frac{1}{2}$ is read as 'What is $\frac{1}{3}$ of $\frac{1}{2}$?' Bar models are used to represent these problems pictorially. Pupils

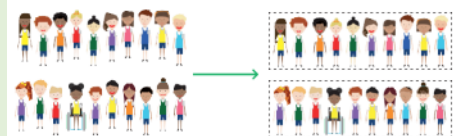
	Multiplying Decimals		progress to realise that the numerators can be multiplied and the denominators can be multiplied, but before this procedure can be embedded, pupils must have a deep understanding of what the equation means. Pupils use the same formal written method procedure as they have previously. Pupils need to pay special attention to the places of the digits in the multiplication. It is important that they do not see the decimal point as a place but rather as a symbol used to separate the whole parts from the decimal parts of a mixed number.
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Division			
Year Group	Strand/Topic	Representation	Key Idea
Reception	Equal Groups		Pupils learn to recognise groups that are equal in quantity, initially using like items and then progressing to different items. Pupils understand that equal groups can be represented by concrete items, diagrams and written numbers. Pupils need to be secure in the abstraction principle of counting the quantity of items regardless of the items' properties or characteristics,

	Subtraction		in order to recognise equal groups in a range of situations. Subtraction and equal groups are concepts that underpin division. During Reception, pupils make equal groups and use equal groups when doubling numbers. While they are doubling numbers, they will see that the whole amount can be partitioned into 2 equal groups.
Year 1	Equal Groups Grouping Sharing	 	Pupils learn to recognise groups that are equal in quantity, initially using like items and then progressing to different items. Pupils understand that equal groups can be represented by concrete items, diagrams and written numbers. Pupils need to be secure in the abstraction principle of counting the quantity of items regardless of the items' properties or characteristics, in order to recognise equal groups in a range of situations. Pupils initially use grouping for division. They put items into equal groups to find the number of equal groups that can be made from a set amount. Pupils move from division through grouping to division through sharing. They share a set amount of items

Division by 2, 5 and 10

20 children can be put into teams of 10.



$20 \div 10 = 2$
There ARE 2 equal teams.

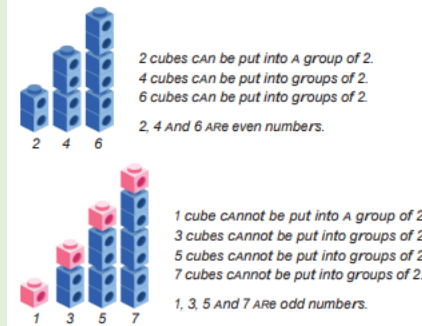
There ARE 2 groups of 10 children.
 $2 \times 10 = 20$

$10 \times 2 = 20$	$20 \div 2 = 10$
$2 \times 10 = 20$	$20 \div 10 = 2$

There is A RELATIONSHIP between the MULTIPLICATION AND division FACTS.

This is A multiplication And division FACT family.

Odd and Even Numbers



2 cubes can be put into A group of 2.
4 cubes can be put into groups of 2.
6 cubes can be put into groups of 2.
2, 4 And 6 Are even numbers.

1 cube cannot be put into A group of 2.
3 cubes cannot be put into groups of 2.
5 cubes cannot be put into groups of 2.
7 cubes cannot be put into groups of 2.
1, 3, 5 And 7 Are odd numbers.

Pupils start to make the connection between division and multiplication. They see amounts as equal groups and relate this to multiplication.

Pupils develop an understanding that even numbers can be put into groups of 2 exactly. Numbers that can be put into groups of 2 and have 1 remaining are described as odd numbers.

Year 3

Dividing by 3, 4 and 8

Sam put 32 cobs of corn into 4 equal groups.



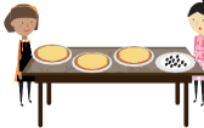
4 groups of 8 is 32.
 $4 \times 8 = 32$

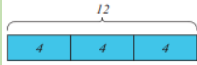
$32 \div 4 = 8$
Each group has 8 cobs of corn.

Pupils are introduced to the division of numbers by 3, 4 and 8 using grouping initially. They make groups of 3, 4 and 8 and then move on to sharing a total.

Division within Word Problems

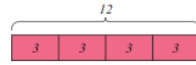
Amira And Ruby Are making pizzas.
They have 12 olives.
They want to put 3 or 4 olives on each pizza.
Can we make A family of multiplication and division equations to help them?





4 times 3 is 12, so
12 divided by 3 is 4.

12 divided into groups
of 4 is EQUAL to 3.



3 times 4 is 12, so
12 divided by 4 is 3.

12 SHARED between
4 is EQUAL to 3.

Pupils extend their understanding of division by relating the division facts to multiplication facts, creating a multiplication and division fact family. Word problems get increasingly more complex and bar models are used to represent problems involving division.

Year 4

Dividing by 6, 7 and 9



$30 \div 6 = 5$
 $6 \times 5 = 30$
Each packet can hold 5 pencils.

When 30 is divided by 6,
the quotient is 5.



Dividing by 11 and 12



$5 \times 12 =$ $\div 12 =$

$12 \times 5 =$ $\div 5 =$

Dividing with remainders

There Are 13 flowers.



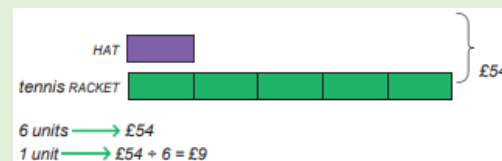
$13 \div 3 = 4$ with 1 left over
The quotient is 4.
The remainder is 1.

Pupils are given division word problems and immediately relate the division used to solve the problem to the multiplication fact they have previously learned. The language associated with division is given, with pupils understanding that when the number is divided, the outcome is called the quotient.

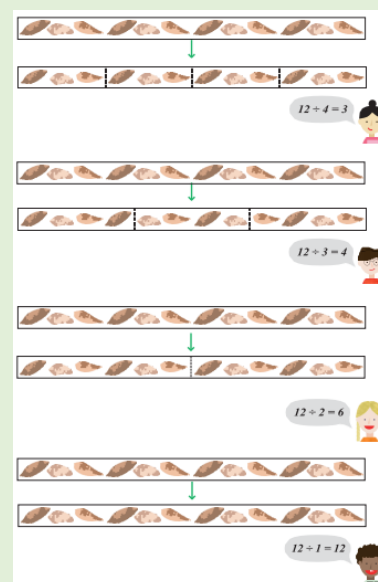
Arrays and bar models are used to show the relationship between multiplication and division when learning to multiply and divide by 11 and 12, building on the relationship already learned when dividing by 6, 7 and 9.

Pupils learn that when dividing into equal groups, we can be left with a number of items less than the group size. This is introduced as the remainder. Initially, the remainder is shown as a whole number.

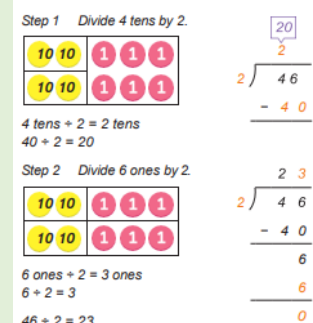
Word Problems Involving Division



Dividing by 1



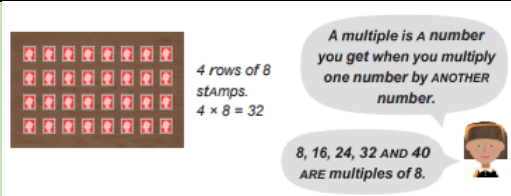
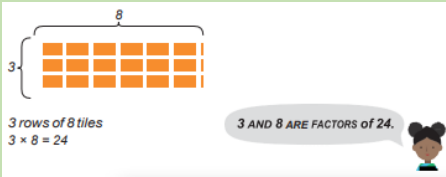
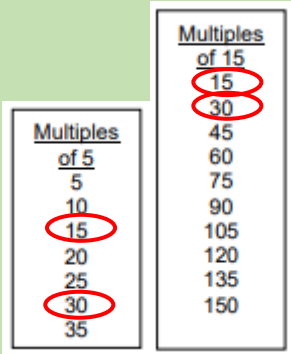
Dividing 2-Digit Numbers



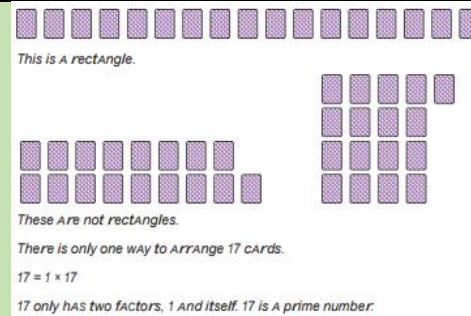
Division word problems are supported by the use of arrays and bar models, reinforcing the idea of equal groups. Pupils relate the representations of the problems to the equations given. comparison division models are also used to determine amounts when two separate amounts are compared.

Pupils look for a pattern and generalise about dividing by 1. They systematically work through dividing a single amount by 4, 3, 2 and finally 1 to make observations about the number of groups and the size of each group.

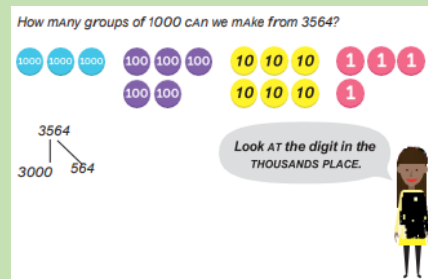
Pupils initially use place-value counters to support the division of 2-digit numbers, then move on to use a long formal written method. The long written method shows the systematic division of parts of the dividend resulting in the quotient.

	Dividing 3-Digit Numbers		The same procedure used for dividing 2-digit numbers is used for dividing 3-digit numbers. Place-value counters are used to represent the problem before moving on to use the long formal written method.
Year 5	Finding Multiples		Pupils use arrays to recognise multiples as the total number once a number is multiplied by another number. Skip counting is related to multiples as it is shown on a number line. Pupils also look for patterns when identifying multiples on number squares. The same rectangular arrangement that was used to find multiples is used to identify factors. The pictorial representation leads to an understanding that factors are the numbers we multiply to produce a product. Pupils learn that when multiple numbers share the same factors, we can describe those factors as common factors. Pupils will begin to generalise about common factors. For example, all whole numbers ending in zero will have 5 as a multiple.
	Finding Factors		
	Finding Common Factors		

Prime and Composite Numbers



Dividing by 10, 100 and 1,000



Dividing without Remainder



Pupils use their understanding of rectangular arrays to look for prime numbers. They learn that any number that can only be made into a single rectangular array is a prime number. In describing this array, they make the connection that prime numbers only ever have two factors, itself and 1. They also learn that numbers with two or more factors can be described as composite numbers.

Place-value counters and number bonds are initially used to represent division problems involving dividing by 10, 100 and 1000. Pupils use their understanding of place value to support the division calculations. For example, 35 hundreds $\div$ 1 hundred = 35.

Pupils use place-value counters and number bond diagrams to support their understanding of the long formal written method for division. Pupils are shown how numbers can be partitioned into known multiples before carrying out the division.

Dividing by a 2-Digit Number with Remainder

Which division method do you prefer?

Common Multiples

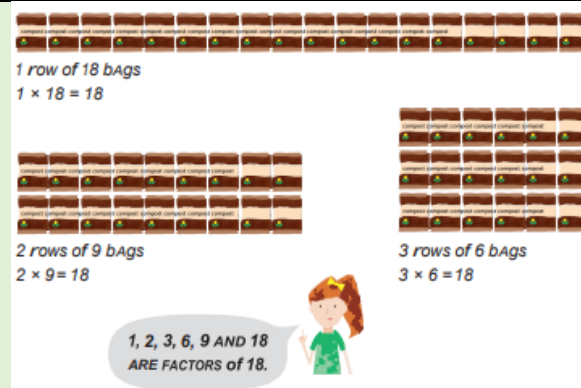
Multiples of 4	4	8	12	16	20	24	28	32	36	40	44	48
Multiples of 6	6	12	18	24	30	36	42	48	54	60	66	72
Multiples of 8	8	16	24	32	40	48	56	64	72	80	88	96

written method. Once the long method is understood, pupils move on to divide using a short formal written method. While the process remains the same, the notation changes to keep it within the short division structure.

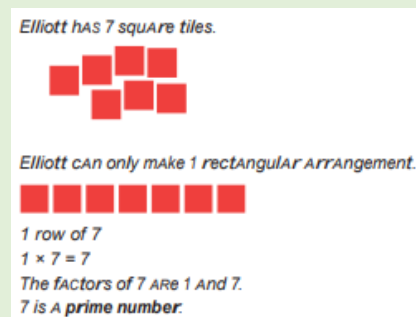
The process used when dividing by a 2-digit number without a remainder stays the same when dividing with remainders. The process results in remainders that cannot be put into the equal group size as whole numbers. The context of the problem suggests the form that the remainder will take and pupils decide on the best representation for the remainder depending on the context. Pupils also use a unitary method of division to solve more complex word problems. Within these problems, they also use brackets to show the partitioning of numbers and how this can be used to support calculation in division problems.

Pupils work systematically through problems looking for common multiples of given numbers.

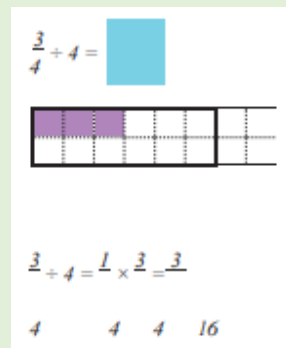
Common Factors



Prime Numbers



Dividing Fractions by Whole Numbers



Pupils use long division to find common factors of given numbers. The method used to find common factors progresses to arrays and using tables to systematically find possible common factors.

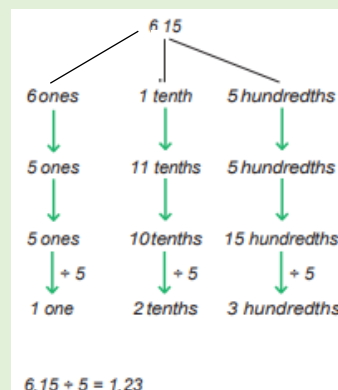
Arrays are used as they have been previously, looking for rectangular patterns. Pupils see that numbers that can only be made into 1 rectangular arrangement are prime numbers with factors of itself and 1.

Pupils relate dividing fractions by a whole number to multiplying by its reciprocal. So, dividing by 4 is related to multiplying by $\frac{1}{4}$. We also read this as ' $\frac{1}{4}$ of'. The procedure of dividing fractions by whole numbers is supported by the use of bar models and pictorial representation.

Dividing Decimals
without Renaming

$$\begin{array}{r}
 2 \overline{) 8.42} \\
 \underline{- 8} \\
 0.4 \rightarrow 2 \times 4 \\
 \underline{- 0.4} \rightarrow 2 \times 0.2 \\
 0.02 \\
 \underline{- 0.02} \rightarrow 2 \times 0.01 \\
 0
 \end{array}$$

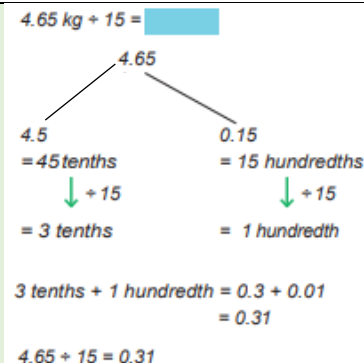
Dividing Decimals with
Renaming



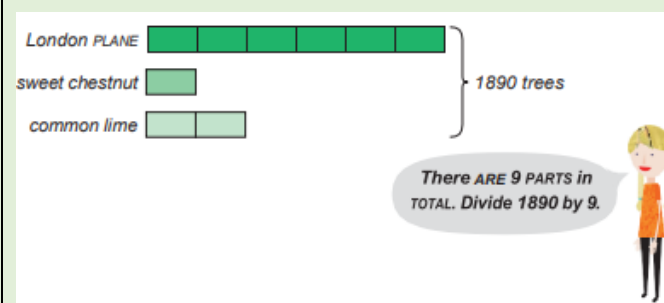
Initially, place-value counters are used to show the division procedure that should be well known by pupils at this stage. The long formal written method is then used to divide decimal numbers without renaming the dividend. The procedure for long division does not change. Pupils need to be mindful of the placement of the digits and remember that the decimal point does not represent a place. Simply, the decimal point separates the whole and fractional parts of a number.

Initially, place-value counters are used to show the division procedure that should be well known by pupils at this stage. The long formal written method is then used to divide decimal numbers without a remainder. The procedure for long division with renaming does not change from what pupils have experienced previously. Pupils need to be mindful of the placement of the digits and remember that the decimal point does not represent a place. Simply, the decimal point separates the whole and fractional parts of a number.

Dividing Decimals by a 2-Digit Whole Number



Ratio



Algebra

x	18	3	90
$\frac{x}{3}$			

Pupils initially divide decimal numbers by 2-digit whole numbers where the dividend is easily broken into multiples of the divisor. Number bonds demonstrate the partitioning in order to divide using long and short formal written methods of division.

Pupils use a unitary method involving division to determine quantities in ratio problems. This approach is supported by the use of bar models.

Pupils use their understanding of division to determine unknown values with algebraic expressions and equations.